

PULSAR RADIO EMISSION CUTOFFS

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Abstract. The propagation of radio emission in pulsar magnetospheres is discussed. We follow a kinematic model in order to derive dispersion relations for electromagnetic oscillations and transversal waves, propagating in a cold moving plasma. We have included relativistic corrections on the dispersion properties, involved with the relativistic motion of the emitting plasma. The occurrence of plasma instabilities is analyzed beside the conditions which should be fulfilled in order to permit the wave propagation and conversion in regions close to the cutoffs of the system. The existence of various frequencies of resonance has been predicted and we are working out these results in order to explain the low-frequency cutoffs observed in radio pulsar spectra.

Keywords: pulsar radio emission, plasma cutoffs, Cherenkov radiation, instabilities

PACS: 97.60.Gb, 94.30.Gm, 95.30.Gv

INTRODUCTION

Many problems in current astrophysics, plasma physics and magnetohydrodynamics are related to the theory of transport radiation through a moving medium. For example, the radiation originated in a neutron star must travel through the moving magnetosphere corotating with the central compact object. Moreover, assuming that the corotation velocity varies with the distance from core, at certain distance from the core the moving medium becomes relativistic and one may expect anomalous behavior from the radiation passing through the plasma.

Recently, several papers have been devoted to study this kind of problem. For instance, [5] pointed out in his review, the processes according to which the magnetospheric plasma, in pulsars, affects propagation of radio emission. There are four main factors that can influence the propagation: reflection, refraction, scattering and absorption. Regarding the absorption, the waves can be absorbed by resonance and scatter in processes involving interaction with plasma particles or another wave. Furthermore, it is supposed that the radio emission is generated in the pulsar outflowing plasma [12] and the propagation effects take place inside the light cylinder. Three main resonances processes have been identified: the Cherenkov resonance [e.g. 1], Cherenkov-drift resonance [e.g. 4] and the cyclotron resonance [e.g. 13, 4]. In order to explain the features of the radio emission observed in pulsars, the above mechanisms beside those corresponding to the transverse electromagnetic waves generation, have been considered. Many papers have been devoted to discuss the kind of electromagnetic waves which are able to propagate in a pulsar plasma, however, the pulsar radio emission models up to now proposed are not able to explain all the main features of observational facts. Furthermore efforts are necessary in order to incorporate, for instance, relativistic effects which may conduct to more detailed descriptions of the absorption and resonance instabilities, which

are supposed to produce electromagnetic waves and oscillations in pulsar plasmas.

In this paper we follow the results of our previous dispersion model [8] developed for relativistic moving magnetoactive plasmas, in order to establish the electromagnetic spectra and the modes corresponding to absorption cutoffs and instabilities, which are propagating in the pulsar magnetosphere as well as in the wind driven by the open magnetic field lines at the polar caps. We show, that the relativistic effect on the dispersion properties of a magnetoactive medium appears to be important when the motion of the plasma occurs with highly relativistic velocities and the gyrofrequency ω_b is of the order of plasma frequency ω_p . The combined action of the magnetic field and bulk velocity increases the influence of the medium on the dispersion processes, which are dependent on the direction of the wave propagation.

In section 1, we describe briefly the dispersion model that we have used and present the plasma solutions that are able to propagate in the pulsar plasma. In section 2, the observational features of the radio emission in pulsar are stated and we interpretate the observations according with the results of the proposed dispersion model.

THE RELATIVISTIC TREATMENT OF AN ANISOTROPIC MAGNETOACTIVE PLASMA

The influence of the global plasma motion on the dispersive properties of the medium has been recently studied by Lopez [8, 6, 7]. Classical treatment of a magnetoactive plasma shows that the global motion does not affect the dispersion properties of the medium. The normal waves should propagate like those in a non-moving system. However, if the plasma is moving at relativistic velocities, its influence on eventual anisotropies turn out to be more substantial and new relativistic effects appear. For instance, the propagation of new electromagnetic modes becomes possible besides the ordinary waves of a magnetoactive plasma [6, 7].

We consider a plasma of low temperature consisting only of electrons and ions, and let both components move relative to each other in equilibrium. Then the low temperature of the plasma makes it possible to treat the particles classically ($\frac{v_t}{c} \ll 1$, where v_t is the thermal velocity). We consider, in addition, the collisionless regime. All of our calculations are translated to the laboratory frame. We begin by finding the suitable dielectric tensor of permittivity ϵ_{ij} for a magnetoactive plasma which is moving with respect to the laboratory frame. Using the Lorentz's transformation, a tensorial relation between the electric vector \vec{E}' in the rest frame and the vector \vec{E} in the laboratory frame can be achieved. A similar relation for the electron currents \vec{j} and \vec{j}' is also settled. With this, we may derive a tensorial relation for the transformation of the tensor of the conductivity σ_{ij} , which is related with the dielectric tensor ϵ_{ij} by the well known expression [2]:

$$\epsilon_{ij}(\omega, \vec{k}) = \delta_{ij} + \frac{4\pi i}{\omega} \sigma_{ij}(\omega, \vec{k}).$$

Following this procedure it is not difficult to derive the relativistic relationship between the elements of the permittivity tensor in different coordinate systems. It is the relativistic generalization of the well known Minkowsky's relation and it is given by [6]:

$$\begin{aligned}
\varepsilon_{ij}(w, \vec{k}) = & \delta_{ij}(1 - \frac{w'^2}{w^2}) + \frac{w'^2}{w^2} \varepsilon'_{ij} - \frac{w'}{w} \gamma (1 - \beta) \frac{\varepsilon'_{iv} v_v v_j}{v^2} \frac{w'}{w^2} \gamma \varepsilon'_{iv} k_v v_j + \\
& \frac{w'}{w} \gamma [(1 - \beta) \frac{v_i v_j}{v^2} - \frac{v_j k_i}{w}] + \frac{w'^2}{w^2} \frac{\gamma (1 - \beta)}{v^2} (v_i \varepsilon'_{\mu j} v_\mu - v_i v_j) \frac{w'}{w} \frac{\gamma^2}{v^4} (1 - \beta)^2 v_i v_j \\
& (v^2 - v_\mu \varepsilon'_{\mu v} v_v) + \frac{w'}{w^2} \frac{\gamma^2 (1 - \beta)}{v^2} v_i v_j (v_\mu \varepsilon'_{\mu v} k_v - v_\mu k_\mu) + \frac{w'}{w^2} \gamma v_i (k'_\mu \varepsilon'_{\mu j} - k'_j) - \\
& \frac{\gamma^2 v_i v_j}{w v^2} (1 - \beta) (k'_\mu \varepsilon'_{\mu v} v_v - v_\mu k'_\mu) + \frac{\gamma^2}{w^2} v_i v_j (k'_\mu \varepsilon'_{\mu v} k_v - k'_\mu k_\mu), \quad (1)
\end{aligned}$$

where $\varepsilon_{ij}(w, \vec{k})$ is the dielectric permittivity tensor in the laboratory frame xyz , $\varepsilon'_{ij}(w', \vec{k}')$ is the dielectric permittivity tensor in the rest frame $x'y'z'$ of the plasma, w is the radiation frequency in the system xyz , w' is the radiation frequency in $x'y'z'$ system, \vec{k} and \vec{k}' are the wave vector in the xyz and $x'y'z'$ systems, respectively, $v(x, y, z)$ is the plasma velocity, and:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \beta = \frac{v}{c}$$

$$w' = \gamma(w - \vec{k} \cdot \vec{v}) \quad \vec{k}' = \vec{k} + \frac{\gamma}{v^2} [(1 - \beta) \vec{k} \cdot \vec{v} - w \frac{v^2}{c^2}] \vec{v}.$$

For the description below, the adopted coordinate system is such that the plasma is flowing along the z -axis parallel to the magnetic field lines, the so-called longitudinal flow, which corresponds to a situation typically observed in astrophysical outflows. Then, the global plasma velocity will be $\vec{v} = v_z \vec{k}$, and the tensor above can be written in the simple matrix form as:

$$\varepsilon = I + \frac{w'}{w} \alpha (\varepsilon' - I) \beta \quad (2)$$

where I is the unitary matrix, ε' is the dielectrical permittivity matrix in the co-moving system and

$$\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\gamma v k'_x}{w'} & 0 & 1 + \gamma[(1 - \beta) + \frac{v k'_z}{w'}] \end{pmatrix}, \quad \beta = \begin{pmatrix} \frac{w'}{w} & 0 & \frac{\gamma v k'_x}{w} \\ 0 & \frac{w'}{w} & 0 \\ 0 & 0 & \frac{w'}{w} - \gamma[(1 - \beta) - \frac{v k'_z}{w}] \end{pmatrix}.$$

The tensor ε'_{ij} for a magnetoactive plasma has the usual form:

$$\varepsilon' = \begin{pmatrix} \varepsilon_\perp & ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \quad (3)$$

where:

$$\varepsilon_{\perp} = 1 - \frac{w_p^2}{w^2 - w_b^2} \quad g = \frac{-w_p^2 w_b}{w(w^2 - w_b^2)} \quad \varepsilon_{\parallel} = 1 - \frac{w_p^2}{w^2}$$

and $w_p = 5.64 \times 10^4 \sqrt{n_e}$ Hz, $w_b = 1.76 \times 10^7 (\frac{B}{\gamma})$ Hz are the plasma frequency and the gyrofrequency, respectively, n_e is the electron density in cm^{-3} , B is the magnetic field in G and γ is the Lorentz factor.

Substituting Eq.(3) into Eq.(2), the dielectrical permittivity tensor for a relativistic moving magnetoactive plasma is derived. This tensor looks like a hermitician matrix for this particular jet configuration, its components are presented here:

$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ -\varepsilon_{xy} & \varepsilon_{xx} & \varepsilon_{yz} \\ \varepsilon_{xz} & -\varepsilon_{yz} & \varepsilon_{zz} \end{pmatrix}, \quad (4)$$

with:

$$\begin{aligned} \varepsilon_{xx} &= -\frac{c^2 k^2 v^2 (w_p^2 - w^2) \cos^2 \theta + 2c^2 k v w (w^2 - w_p^2) \cos \theta}{w^2 [c^2 k^2 v^2 \cos^2 \theta - 2c^2 k v w \cos \theta + c^2 (w^2 - w_b^2) + w_b^2 v^2]} \\ &\quad + \frac{w^2 [c^2 (w_b^2 + w_p^2 - w^2) - w_b^2 v^2]}{w^2 [c^2 k^2 v^2 \cos^2 \theta - 2c^2 k v w \cos \theta + c^2 (w^2 - w_b^2) + w_b^2 v^2]} \\ \varepsilon_{zz} &= 1 + \frac{w_p^2 v^2}{c^2 (k v \cos \theta - w)^2} - \frac{w_p^2}{(k v \cos \theta - w)^2} - \\ &\quad \frac{c^2 k^2 w_p^2 v^2 \sin^2 \theta}{w^2 [c^2 k^2 v^2 \cos^2 \theta - 2c^2 k v w \cos \theta + c^2 (w^2 - w_b^2) + w_b^2 v^2]} \\ \varepsilon_{xy} &= \frac{i c w_b w_p^2 \sqrt{c^2 - v^2} (k v \cos \theta - w)}{w^2 [c^2 k^2 v^2 \cos^2 \theta - 2c^2 k v w \cos \theta + c^2 (w^2 - w_b^2) + w_b^2 v^2]} \\ \varepsilon_{xz} &= \frac{c^2 k w_p^2 v \sin \theta (k v \cos \theta - w)}{w^2 [c^2 k^2 v^2 \cos^2 \theta - 2c^2 k v w \cos \theta + c^2 (w^2 - w_b^2) + w_b^2 v^2]} \\ \varepsilon_{yz} &= \frac{i c k w_b w_p^2 v \sqrt{c^2 - v^2} \sin \theta}{w^2 [c^2 k^2 v^2 \cos^2 \theta - 2c^2 k v w \cos \theta + c^2 (w^2 - w_b^2) + w_b^2 v^2]} \end{aligned}$$

Using the Maxwell's equations together with the material equation $D_i = \varepsilon_{ij} E_j$, and adopting the assumption that the perturbation has the form $\exp i(\vec{k} \cdot \vec{r} - \omega t)$, the algebraic equations that describe the wave dispersion in a plasma can be derived and expressed as:

$$[k^2 \delta_{ij} - k_i k_j - \frac{w^2}{c^2} \varepsilon_{ij}(w, \vec{k})] \vec{E}_j = 0$$

The condition to solve this system of equations defines the electromagnetic waves which are allowed to propagate inside a moving magnetoactive plasma. This condition,

with the application of some determinant properties, may easily be converted into the following tensorial dispersion equation [see 8, 6, 7]:

$$\frac{k_i k_j \epsilon_{ij}}{k^2} k^4 + \left[\frac{k_i k_j}{k^2} \epsilon_{i\mu} \epsilon_{\mu j} - \epsilon_{ii} \frac{k_i k_j \epsilon_{ij}}{k^2} \right] \left[\frac{w^2}{c^2} k^2 \right] + |\epsilon_{ij}| \frac{w^4}{c^4} = 0. \quad (5)$$

Now, we must use Eq.(5) along with the dielectric tensor of permittivity given by Eq.(4), in order to derive the electromagnetic spectrum characteristic of a moving magnetoactive plasma. The dispersion Eq.(5) may be solved for an arbitrary dispersion angle ψ , that is, the angle between the jet velocity and the direction of the radiation propagation defined by the wave vector \vec{k} [6, 7]. This is a large and complex tenth order equation in w containing 123 terms, which does not admit exact solutions except for some particular field-velocity orientations.

The above formulation can be applied to any magnetoactive plasma which is moving parallel to the magnetic field. For these systems, we will notice that the higher the relativistic velocity of the plasma the more important the relativistic effects over the dispersive properties are.

In what follows, we discusse the longitudinal modes, these ones have refraction indices $n \gg 1$ and excite pressure waves and turbulence in the plasma. The other cutoffs are contained in the transverse electromagnetic normal modes, disussed in a separated paper [e.g. 8].

Longitudinal wave propagation

In this case we considere that the electromagnetic radiation is propagating along the magnetic field, then the permissible transverse electromagnetic modes that could propagate in a magnetoactive moving plasma are described by the following dispersion relation, which is derived analytically [e.g. 8]:

$$n_{1,2}^2 = 1 \pm \frac{w_p^2 \left[\frac{w_b}{\gamma} \mp \left(1 - \frac{v}{c} \right) w \right]}{w \left[\left(1 - \frac{v}{c} \right) w^2 - \left(1 + \frac{v}{c} \right) w_b^2 \right]} \quad (6)$$

The corresponding curves are similar to the dispersion curves for waves propagating along the magnetic field lines in a static magnetoactive plasma. But in a moving plasma the extraordinary branch, have a cutoff near the gyrofrequency (w_b) with a strong dependence on the plasma velocity:

$$w_r = \left(1 + \frac{v}{c} \right) \gamma w_b. \quad (7)$$

For small velocities $\frac{v^2}{c^2} \ll 1$, the above expression falls into the classical ordinary and extraordinary solutions for a non-moving plasma [2]:

$$n_1^2 = 1 - \frac{w_p^2}{w(w + w_b)} \quad n_2^2 = 1 - \frac{w_p^2}{w(w - w_b)}. \quad (8)$$

The other analytical solution of the dispersion equation corresponds to the longitudinal waves which are described by:

$$n_{3,4} = \frac{c}{v} \left[1 \pm \frac{w_p}{w} \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \right] \quad (9)$$

Therefore, for the radiation propagation along the magnetic field, we have identified two cutoffs: one at the frequency $\frac{w_b}{\gamma}$ and the second given by the Cherenkov-like resonance $w - \vec{k} \cdot \vec{v} - \frac{w_p}{\gamma} = 0$. At this point, it is worthy to note that the latest Doppler-shifted resonance appears as if the magnetic field is not present. This is in contrast, to that stated in the literature, where a hybrid Cherenkov-cyclotron resonance has been derived from a magnetohydrodynamic point of view [4, 9].

Transverse wave propagation

For the case when the wave is propagating perpendicular to the direction of the magnetic field ($\psi = 90^\circ$), the Doppler effect becomes quadratic and the radiation is characterized by the following analytic dispersion relations:

$$n_{1,2}^2 = \left(1 - \frac{w_p^2}{w^2}\right) \pm \frac{\frac{w_b}{w} \frac{w_p^2}{w^2} \left[\sqrt{\frac{w_b^2}{w^2} + 4 \frac{v^2}{c^2} \left(1 - \frac{w_b^2}{w^2} - \frac{w_p^2}{w^2}\right)} - \frac{w_b}{w} \right]}{2 \left[\left(1 - \frac{w_b^2}{w^2} - \frac{w_p^2}{w^2}\right) + \frac{v^2}{c^2} \left(\frac{w_b^2}{w^2} + \frac{w_p^2}{w^2}\right) \right]}. \quad (10)$$

In this case, the frequencies for the points of resonance are given by:

$$w_r = 0, \quad w_r = \frac{\sqrt{(w_b^2 + w_p^2)}}{\gamma}. \quad (11)$$

Also here, we touch only the existence of vacuum-like resonance, i.e. electromagnetic waves that may leave freely the magnetized plasma. Furthermore, as we must expect, in the transverse direction we do not have any longitudinal oscillation [6, 7]. Consequently, in this direction we do not have instabilities due to the electron-wave interaction. The analytical solutions from the general dispersion equation (4), produce again two different electromagnetic branches. One of these solutions corresponds to an ordinary wave (n_1^2) and the other one to an extraordinary wave (n_2^2).

Under the assumption that the outflow moves slowly (classical approach), we have different points of oscillation that may develop instabilities. For instance, considering that the cold magnetoactive plasma is moving along the magnetic field, we have found that the Cherenkov-like plasma oscillations are splitted into the cyclotron-like and plasma frequency-like, i.e., these instabilities are desconnected:

$$w - \vec{k} \cdot \vec{v} - w_b = 0 \quad w - \vec{k} \cdot \vec{v} - w_p = 0. \quad (12)$$

On the other hand, if the plasma is moving transverse to the magnetic field lines, the cyclotron and plasma instabilities appear connected and cyclotron plasma Cherenkov-like oscillations are developed:

$$\omega - \vec{k} \cdot \vec{v} - \omega_p - \omega_b = 0 \quad (13)$$

Furthermore, it is feasible to show that Cherenkov-like instabilities also appear for an arbitrary direction of the waves. However, in the general case, the related expressions are more complex and the dispersion relations and cutoffs should be obtained applying numerical methods.

PULSAR RADIO EMISSION

Owing to energetic charged particles which are produced inside the light cylinder, close to the core, the pulsars are characterized by a radiating magnetosphere. The plasma instabilities seem to be responsible for developing relativistic particle beam, propagating along the open field lines to merge into powerful pulsar wind [e.g. 3]. The wind is moving relativistically at the light cylinder region, whereas the corotating magnetosphere is also moving with highly velocities. Then, a dispersion model taking into account the bulk motion of these media could be able to suggest how the radiation, that is passing through, is affected.

On the other hand, the features of normal modes also should be predicted by such a dispersion model. Therefore, the low-frequency cutoffs observed between 100 MHz and 1 GHz in radio pulsar spectra [e.g. 10, 11], may be physically explained. The occurrence of low-frequency cutoffs observed in the profiles of the pulsar emission are believed to take place in the magnetized plasma of the pulsar magnetosphere. Equations (7) and (11) predict the frequency cutoffs for radio emission coming from these regions. In both expressions, the cyclotron ω_b and plasma ω_p frequencies (natural plasma frequencies), have been attenuated by the plasma bulk γ -factor. We note also that the appropriate cutoff will take place in dependence of magnetic field orientation. For the inner regions toward the core, the cyclotron resonance is most probable (see Eq. 7), whereas beyond the light cylinder the hybrid ω_b and ω_p resonance, given by the equation (11), is more feasible. For estimates, we assume the typical plasma parameters at the emission region, $R \sim 10^9 \text{ cm}$ [4]. At the plasma rest frame the cyclotron frequency $\omega_b = 1.8 \times 10^{10} \text{ rad s}^{-1}$ and the plasma frequency $\omega_p = 1.2 \times 10^8 \text{ rad s}^{-1}$. Then, the corresponding frequency cutoff attenuated by plasma bulk $\gamma = 10$ should be $\nu_r \sim 0.3 \text{ GHz}$, value of the order of the typical frequency of emission, $\sim 1 \text{ GHz}$, of pulsars [11]. For the inner regions at the surface of the neutron star, the equation (7) gives us a much higher frequency cutoff $\nu_r \sim 2.9 \times 10^{17} \text{ Hz}$; assuming a dipolar magnetic field of 10^{12} G . Therefore, we conclude that the observed radiation probably is generated in external emitting regions at $R \sim 10^9 \text{ cm}$ from the core, in agreement with previous works [e.g. 4].

On the other hand, the equations (9), (12) and (13) give us the existence in the moving plasma of Cherenkov-like oscillations. Physically, these instabilities represent Doppler-shifted resonances and generally reveal flow advolutions. However, these plasma

instabilities are able to develop radiation by maser-type mechanics, producing induced electromagnetic radiation. The Cherenkov-like instabilities typically occur in the outer magnetosphere and can be handled in order to explain some observational facts present in the pulsar radio profiles.

ACKNOWLEDGMENTS

The author gives many thanks to the Brazilian agencies CLAF and CNPq that supported partially this work and the hospitality of the Instituto Astronômico e Geofísico, IAG-USP, where part of this paper was discussed previously.

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